

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] MAY-JUNE 2026

Paper Code: BS-1.12

Subject: Applied Mathematics-II

Time: 3 Hours

Maximum Marks: 60

Note: Attempt all questions as directed. Internal choice is indicated.

Q1 Attempt any four of the following questions:

(4x5=20)

- (a) Determine the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$
- (b) Evaluate $\int_0^{3+i} z^2 dz$ along the parabola $y^2 = \frac{x}{3}$.
- (c) State and prove Second shifting property of Laplace Transformation.
- (d) Show that the function $f(z) = z|z|$ is not analytic anywhere.
- (e) Find the linear fractional transformation (Mobius transformation) which maps the points $z = 0, 1, \infty$ to the points $w = -1, -i$ and i respectively.
- (f) Obtain Laplace transform of $f(t) = t^2 e^t \sin t$.
- (g) Find the Fourier cosine transform of $f(x) = e^{-x} + e^{-2x}$, $x > 0$
- (h) Form a Partial Differential Equation from $z = f\left(\frac{x}{z}\right) + g\left(\frac{y}{z}\right)$.
- (i) Determine the poles of the function $f(z) = \frac{z}{(z-1)(z-2)^2}$ and the residue at each pole.
- (j) Prove that the function $f(z) = \frac{z^3(1+i)-y^3(1-i)}{x^2+y^2}$, $z \neq 0$ and $f(0) = 0$ is continuous and Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

OR

- Q3 (a) Evaluate $\int_C \frac{dz}{(z-1)(z-2)(z-3)}$ where C is the circle $|z| = 4$ (5)
- (b) Find the Laurent series of $f(z) = \frac{7z-2}{z^3-z^2-2z}$ in the region (i) $0 < |z+1| < 1$ (5)
- (ii) $1 < |z+1| < 3$ (5)

- Q4 (a) Obtain the condition under which the mapping $w = \frac{az+b}{cz+d}$ maps a straight line of z -plane into a unit circle of w -plane. (5)
- (b) Using residue theorem, evaluate $\int_C \frac{e^{z^2} dz}{(z+1)^2(z-2)}$ where $C: |z-1| = 3$ (5)

OR

- Q5 (a) An electrostatic field in the xy -plane is given by the potential function $\phi = 3x^2y - y^3$. Find the Stream function. (5)
- (b) Evaluate $\int_0^\pi \frac{a \sin \theta}{a^2 + \sin^2 \theta} d\theta$ ($0 < a < 1$) (5)

- Q6 (a) Find the Laplace transform of (i) $\left[\frac{\cos at - \cos bt}{t} \right]$, (ii) $\frac{e^{t^2}}{(t^2-4t+13)}$ (5)
- (b) Solve, using Laplace Transform technique, the differential equation $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 5y = e^{-t} \sin t$ where $y(0) = 0$, $y'(0) = 1$ (5)

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- Q7 (a) Obtain the Fourier series for the function $f(x)$ given by

OR

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & \text{for } -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & \text{for } 0 \leq x \leq \pi \end{cases}$$

(5)

- (b) Using Fourier transform to solve the equation $\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2}$, $x > 0$, $t > 0$ subject to the conditions (i) $u(0, t) = 0$, $t > 0$ (ii) $u(x, 0) = 1$, $0 < x < 1$ (iii) $u(x, t)$ is bounded. (5)

- (a) Using the method of separation of variables, solve the equation $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ where $u(x, 0) = e^{-3x}$ (5)

- (b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = a \sin\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, show that the displacement of any point at a distance x from one end at time t is given by

$$y(x, t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi c t}{l}\right).$$

(5)

OR

- Q9 (a) A rod of length l with insulated sides is initially at a uniform temperature u_0 . Its ends are suddenly cooled to $0^\circ C$ and are kept at that temperature. Find the temperature function $u(x, t)$. (5)

- (b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary condition the rectangle $u(x, 0) = 3 \sin \pi x$, $u(0, t) = 0$, and $u(l, t) = 0$ where $0 < x < l$, $t > 0$ (5)

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